HEAT TRANSPORT I. Heat balance, steam table

Problem 1

Water stream 100 kg/h, 20°C is warmed up.

a) What is the final temperature if it is mixed with a water stream 300 kg/h, 80°C?
b) How much steam of pressure 1.8 bar and humidity 3 % is to be mixed with it to get 70°C water?

Solution

a) What is the final temperature if it is mixed with a water stream 300 kg/h, 80°C?

h' in the steam table is the specific enthalpy of the water just boiling at the given temperature. This value is valid under the vapor pressure of the water at this temperature. At temperatures smaller than 100 C this is smaller than the atmospheric pressure.
However, this enthalpy can be taken as a very good approximation of the actual enhalpy under boiling point but at atmospheric pressure. The liquid enthalpy mainly depends on the temperature only, not the pressure, because of its incompressibility.

h'_{20} = 83.903 kJ/kg h'_{80} = 334.944 kJ/kg

Heat balance

$$\sum \dot{Q}_{in} = \sum \dot{Q}_{out}$$

$$\dot{m}_{20} \cdot h'_{20} + \dot{m}_{80} \cdot h'_{80} = \left(\dot{m}_{20} + \dot{m}_{80}\right) \cdot h'_{?}$$

$$100\frac{kg}{h} \cdot 83.903\frac{kJ}{kg} + 300\frac{kg}{h} \cdot 334.944\frac{kJ}{kg} = \left(100\frac{kg}{h} + 300\frac{kg}{h}\right) \cdot h'_{?}$$

$$h'_{?} = 272.2\frac{kJ}{kg}$$

The unknown temperature can be read out from the steam table or can be calculated with average specific heat.
The two method gives approximately the same results.

$$\Delta T = \frac{h'_{?}}{c_p} = \frac{272.2\frac{kJ}{kg}}{4.18\frac{kJ}{kg \cdot K}} = 65.11°C$$

This is a temperature difference only. If the reference point of the enthalpy table is the freezing point (i.e. $h'_0 = 0\frac{kJ}{kg}$) then $T_{out} = 65.11°C$.

Reading the steam table results:
h'_{65} = 272,058 kJ/kg h'_{66} = 276,245 kJ/kg

Linear interpolation:

$$\frac{h'_? - h'_{65}}{h'_{66} - h'_{65}} = \frac{T - 65°C}{66°C - 65°C}$$

$$T = \frac{h'_? - h'_{65}}{h'_{66} - h'_{65}} \cdot (66°C - 65°C) + 65°C = \frac{272.2\frac{kJ}{kg} - 272.058\frac{kJ}{kg}}{276.245\frac{kJ}{kg} - 272.058\frac{kJ}{kg}} \cdot 1°C + 65°C = 65.03°C$$

Interpolation in the steam table provides results within 0.1 C.

b) How much steam of pressure 1.8 bar and humidity 3 % is to be mixed with it to get 70°C water?

$p_{St} = 1.8 \cdot 10^5$ Pa
$x_{St} = 0.97$ (fraction of dry steam)
T' = 70°C
$\dot{m}_{St} = ?$

Wet steam is normally formed because of condensation due to heat loss.
When steam flow rate is considered, the condensate is included in the flow rate, it is part of the stream.

From the steam table:
$T_{St} = 117°C$ $h'_G = 490.986$ kJ/kg $h''_{St} = 2702.161$ kJ/kg
$h'_{70} = 292.992$ kJ/kg

Heat balance:

$$\dot{m}_{20} \cdot h'_{20} + \dot{m}_{St} \cdot \left((1 - x_{St}) \cdot h'_{St} + x_{St} \cdot h''_{St}\right) = \left(\dot{m}_{20} + \dot{m}_{St}\right) \cdot h'_{70}$$

$$100\frac{kg}{h} \cdot 83.903\frac{kJ}{kg} + \dot{m}_{St} \cdot \left((1 - 0.97) \cdot 490.986\frac{kJ}{kg} + 0.97 \cdot 2702.161\frac{kJ}{kg}\right) =$$

$$= \left(100\frac{kg}{h} + \dot{m}_{St}\right) \cdot 292.992\frac{kJ}{kg}$$

$$2342.83\frac{kJ}{kg} \cdot \dot{m}_{St} = 20908.9\frac{kJ}{h}$$

$$\dot{m}_{St} = 8.92\frac{kg}{h}$$

Problem 2

Superheated steam of 10^6 Pa and 300°C is to be changed to wet steam of dry steam content x = 0.9 under the same pressure by spraying in water of 50 C. How much water is to be sprayed in to 1 kg steam?

Data:

10^6 Pa 300°C steam: h_1 = 3052.2 kJ/kg

10^6 Pa saturated steam h_2'' = 2778.1 kJ/kg ΔH_2^{vap} = 2105.6 kJ/kg

Solution

Steam table

50°C water: h_3' = 209.298 kJ/kg

Heat balance

$$m_1 \cdot h_1 + m_3 \cdot h'_3 = (m_1 + m_3) \cdot \left[h''_2 - (1 - x) \cdot \Delta H_2^{vap}\right]$$

$$1kg \cdot 3052.2\frac{kJ}{kg} + m_3 \cdot 209.298\frac{kJ}{kg} = (1kg + m_3) \cdot \left[2778.1\frac{kJ}{kg} - (1 - 0.9) \cdot 2105.6\frac{kJ}{kg}\right]$$

$$3052.2kJ + m_3 \cdot 209.298\frac{kJ}{kg} = (1kg + m_3) \cdot 2567.54\frac{kJ}{kg}$$

$$3052.2kJ + m_3 \cdot 209.298\frac{kJ}{kg} = 2567.54kJ + m_3 \cdot 2567.54\frac{kJ}{kg}$$

$$484.66kJ = m_3 \cdot 2358.242\frac{kJ}{kg}$$

$$m_3 = 0.21kg$$

Problem 3

60°C warm water is to be mixed from 100 kg/h 20°C water, 300 kg/h 40°C water, and 120°C saturated steam. How much steam is needed?

Solution

Steam table:

$h_{20}' = 83.903$ kJ/kg
$h_{40}' = 167.514$ kJ/kg
$h_{60}' = 251.124$ kJ/kg
$h_{120}'' = 2706.348$ kJ/kg

Heat balance:

$$\dot{m}_{20}\cdot h'_{20}+\dot{m}_{40}\cdot h'_{40}+\dot{m}_{120}\cdot h''_{120}=\left(\dot{m}_{20}+\dot{m}_{40}+\dot{m}_{120}\right)\cdot h'_{60}$$

$$100\frac{kg}{h}\cdot 83.903\frac{kJ}{kg}+300\frac{kg}{h}\cdot 167.514\frac{kJ}{kg}+\dot{m}_{120}\cdot 2706.348\frac{kJ}{kg}=$$

$$=\left(100\frac{kg}{h}+300\frac{kg}{h}+\dot{m}_{120}\right)\cdot 251.124\frac{kJ}{kg}$$

$$58644.5\frac{kJ}{h}+\dot{m}_{120}\cdot 2706.348\frac{kJ}{kg}=100449.6\frac{kJ}{h}+\dot{m}_{120}\cdot 251.124\frac{kJ}{kg}$$

$$\dot{m}_{120}\cdot 2455.224\frac{kJ}{kg}=41805.1\frac{kJ}{h}$$

$$\dot{m}_{120}=17.03\frac{kg}{h}$$

HEAT TRANSPORT II. Multilayer walls

Heat transport types

λ heat conductivity $\left[\frac{W}{m \cdot K}\right]$ [wall] or [fluid bed]

α heat transport coefficient (film) $\left[\frac{W}{m^2 \cdot K}\right]$ [fluid to wall] or [wall to fluid]

U overall heat transport coefficient $\left[\frac{W}{m^2 \cdot K}\right]$ [fluid 1 to wall] & [wall] & [wall to fluid 2]

There is also heat radiation.

Multilayer wall

Thermal resistances of the layers are added:

$$\frac{1}{R} = \frac{1}{\sum_i R_i}$$

Overall heat transport

Considering n solid (planar) layers with width w :

$$\frac{1}{U} = \frac{1}{\alpha_1} + \sum_{i=1}^{n} \frac{w_i}{\lambda_i} + \frac{1}{\alpha_2}$$

Heat conduction

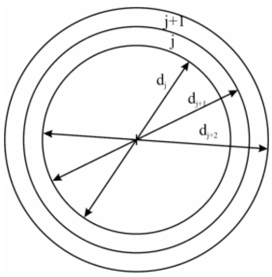
λ heat conductivity $\left[\frac{W}{m \cdot K}\right]$

Throught planar wall:

$$\dot{Q} = \frac{1}{\sum_j \frac{w_j}{\lambda_j}} \cdot A \cdot (T_1 - T_2)$$

where w is width [m]

Through circular wall



$$\dot{Q} = \frac{2 \cdot \pi \cdot L}{\sum_j \frac{1}{\lambda_j} \cdot \ln \frac{d_{j+1}}{d_j}} \cdot (T_1 - T_2)$$

where L tube length [m]

d diameter [m]

Outer diameter containes wall thickness twice!

Heat resistance

$$R_j = \frac{w_j}{\lambda_j} \qquad \left[\frac{m^2 \cdot K}{W}\right]$$

Problem 4

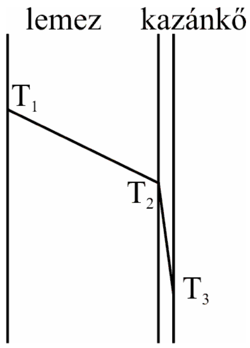
1 mm scaling layer is formed on the inner side of a 20 mm iron boiler plate. Temperature at the outer side of the wall is 600 C, at the inner side it is 240 C. Heat conductivity of iron is 58 W/mK, that of scaling is 1.2 W/mK.

Calculate:

a) Heat flux without scaling
b) Heat flux with scaling
c) Temperature between the iron plate and the scaling
d) How many times is the thermal resistance with scaling than without?

<u>Solution</u>

w_l = 20 mm
w_k = 1 mm
T_1 = 600°C
T_2 = 240°C
λ_l = 58 W/mK
λ_k = 1.2 W/mK



lemez: plate
kazánkő: scaling

a) Heat flux without scaling

$$\left(\frac{\dot{Q}}{A}\right)_1 = \frac{1}{\frac{w_l}{\lambda_l}} \cdot (T_1 - T_3) = \frac{1}{\frac{0.02\text{m}}{58\frac{\text{W}}{\text{m}\cdot\text{K}}}} \cdot (600°\text{C} - 240°\text{C}) = 1.04 \cdot 10^6 \frac{\text{W}}{\text{m}^2}$$

b) Heat flux with scaling

$$\left(\frac{\dot{Q}}{A}\right)_2 = \frac{1}{\sum_j \frac{w_j}{\lambda_j}} \cdot (T_1 - T_3) = \frac{1}{\frac{0.02\text{m}}{58\frac{\text{W}}{\text{m}\cdot\text{K}}} + \frac{0.001\text{m}}{1.2\frac{\text{W}}{\text{m}\cdot\text{K}}}} \cdot (600°\text{C} - 240°\text{C}) = 3.056 \cdot 10^5 \frac{\text{W}}{\text{m}^2}$$

The flux drops due to scaling to 30 % of the case without scaling.

c) Temperature between the iron plate and the scaling

c/1 Calculation through the plate

$$\left(\frac{\dot{Q}}{A}\right)_2 = \frac{1}{\frac{w_{plate}}{\lambda_{plate}}} \cdot (T_1 - T_2)$$

$$T_2 = T_1 - \left(\frac{\dot{Q}}{A}\right)_2 \cdot \frac{w_l}{\lambda_l} = 600°\text{C} - 3.056 \cdot 10^5 \frac{\text{W}}{\text{m}^2} \cdot \frac{0{,}02\text{m}}{58\frac{\text{W}}{\text{m}\cdot\text{K}}} = 494.6°\text{C}$$

c/2 Calculation through the scaling

$$\left(\frac{\dot{Q}}{A}\right)_2 = \frac{1}{\frac{w_{scaling}}{\lambda_{scaling}}} \cdot (T_2 - T_3)$$

$$T_2 = T_3 + \left(\frac{\dot{Q}}{A}\right)_2 \cdot \frac{s_{plate}}{\lambda_{plate}} = 240°C + 3.056 \cdot 10^5 \frac{W}{m^2} \cdot \frac{0,001m}{1.2\frac{W}{m \cdot K}} = 494.7°C$$

Results are within rounding error.

Note:
This explanes many boiler explosions. Without scaling the water touches the 240 C plate surface.
If there is scaling then water touches the 240 C scaling surface, but if the scaling ruptures then the water touches the almost 500 C plate and boils up suddenly.

d) How many times is the thermal resistance with scaling than without?

Resistance of the iron plate:

$$R_1 = \frac{w_1}{\lambda_1} = \frac{0.02m}{58\frac{W}{m \cdot K}} = 3.45 \cdot 10^{-4} \frac{m^2 \cdot K}{W}$$

Resistance of the scaling layer:

$$R_{sc} = \frac{w_{sc}}{\lambda_{sc}} = \frac{0.001m}{1.2\frac{W}{m \cdot K}} = 8.33 \cdot 10^{-4} \frac{m^2 \cdot K}{W}$$

Ratio of the resistances:

$$x = \frac{R_{sc} + R_1}{R_1} = \frac{8.33 \cdot 10^{-4} \frac{m^2 \cdot K}{W} + 3.45 \cdot 10^{-4} \frac{m^2 \cdot K}{W}}{3.45 \cdot 10^{-4} \frac{m^2 \cdot K}{W}} = 3.4$$

Note: Ratio of layer resistances equals to ratio of temperature drops over these layers:

$$\frac{R_{sc}}{R_1} = \frac{8.33 \cdot 10^{-4} \frac{m^2 \cdot K}{W}}{3.45 \cdot 10^{-4} \frac{m^2 \cdot K}{W}} = \frac{T_2 - T_1}{T_3 - T_2} = \frac{494.6°C - 240°C}{600°C - 494.6°C} \approx 2.42$$

Problem 5

Temperature inside of a steel tube (diameters 30/20 mm, λ = 17.4 W/mK) is 600°C, outside it is 450°C.
What is the heat power over a tube section of 1 m length?

Solution

$$\dot{Q} = \frac{2 \cdot \pi \cdot L}{\frac{1}{\lambda} \cdot \ln\frac{d_{out}}{d_{in}}} \cdot (T_1 - T_2) = \frac{2 \cdot \pi \cdot 1m}{\frac{1}{17.4\frac{W}{m \cdot K}} \cdot \ln\frac{30mm}{20mm}} \cdot (600°C - 450°C) = 40.44kW$$

Problem 6

A steam pipeline with outer diameter 100 mm is covered with two insulating layers, both are 25 mm wide. Heat conductivity of the first one is 0.070 W/mK, the other is 0.087 W/mK. Outside wall temperature is 200°C; outer temperature is 40°C.

a) How much is the heat loss over a section of 1 m lenght?
b) What is the temperature between the two insulating layers?

Solution

a) How much is the heat loss over a section of 1 m lenght?

Diameters:

$$d_2 = d_1 + 2 \cdot w_1 = 100\text{mm} + 2 \cdot 25\text{mm} = 150\text{mm}$$
$$d_3 = d_2 + 2 \cdot w_2 = 150\text{mm} + 2 \cdot 25\text{mm} = 200\text{mm}$$

Heat loss

$$\dot{Q} = \frac{2 \cdot \pi \cdot L}{\sum_j \frac{1}{\lambda_j} \cdot \ln \frac{d_{j+1}}{d_j}} \cdot (T_1 - T_3)$$

$$\dot{Q} = \frac{2 \cdot \pi \cdot 1\text{m}}{\frac{1}{0.07 \frac{W}{m \cdot K}} \cdot \ln \frac{150\,\text{mm}}{100\,\text{mm}} + \frac{1}{0.087 \frac{W}{m \cdot K}} \cdot \ln \frac{200\,\text{mm}}{150\,\text{mm}}} \cdot (200°C - 40°C) = 110.5\text{W}$$

b) What is the temperature between the two insulating layers?

Counting from inside

$$\dot{Q} = \frac{2 \cdot \pi \cdot L}{\frac{1}{\lambda_1} \cdot \ln \frac{d_2}{d_1}} \cdot (T_1 - T_2)$$

$$T_2 = T_1 - \frac{\dot{Q} \cdot \frac{1}{\lambda_1} \cdot \ln \frac{d_2}{d_1}}{2 \cdot \pi \cdot L} = 200°C - \frac{110.5\text{W} \cdot \frac{1}{0.07 \frac{W}{m \cdot K}} \cdot \ln \frac{150\text{mm}}{100\text{mm}}}{2 \cdot \pi \cdot 1\text{m}} = 98.13°C$$

Counting from outside

$$\dot{Q} = \frac{2 \cdot \pi \cdot L}{\frac{1}{\lambda_2} \cdot \ln \frac{d_3}{d_2}} \cdot (T_2 - T_3)$$

$$T_2 = \frac{\dot{Q} \cdot \frac{1}{\lambda_2} \cdot \ln \frac{d_3}{d_2}}{2 \cdot \pi \cdot L} + T_3 = \frac{110.5\text{W} \cdot \frac{1}{0.087 \frac{W}{m \cdot K}} \cdot \ln \frac{200\text{mm}}{150\text{mm}}}{2 \cdot \pi \cdot 1\text{m}} + 40°C = 98.15°C$$

HEAT TRANSPORT III. Heat radiation

$$\dot{Q} = \varepsilon \cdot C_0 \cdot A \cdot \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]$$

where ε emissivity (degree of blackness) [–]

C_0 10^8 multiple of the Stefan-Boltzmann constant σ_0 $5.67 \frac{W}{m^2 \cdot K^4}$

($100^4 = 10^8$ and counting this way is easier.)

Radiation heat transport coefficient is defined with analogy to heat transport from wall to fluid:

$$\dot{Q} = \alpha_{rad} \cdot A \cdot (T_1 - T_2)$$

where α_{rad} radiation heat transport coefficient $\left[\frac{W}{m^2 \cdot K} \right]$

Heat radiation is usually accompanied by heat transport from wall to fluid.

Problem 7

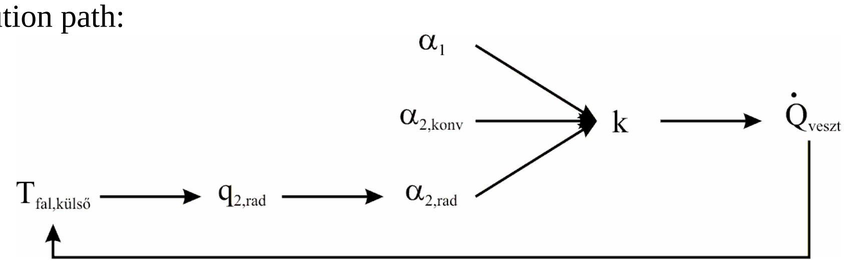
Drying is performed in an oven at 105°C. Oven wall is 2 mm wide, its heat conductivity is 58 W/mK, its emissivity is 0.9. Heat transport coefficient from inside tho oven to the wall is 1300 W/m^{2}K, outside from the wall to the air is 9 W/m^{2}K.

a) How much is the heat loss to the 20°C environment over 1 m^2 of the oven wall?
b) 1 cm wide insulation is put around the oven due to safety reasons. Its heat conductivity is 0.07 W/mK, its emissivity is 0.75. What will be the outside surface temperature?

Solution

a) How much is the heat loss to the 20°C environment over 1 m^2 of the oven wall?

Solution path:



külső: outside veszt: loss

The iron wall is a good conductor, and the air is not, the internal temperature can be taken as a good initial temperature estimate.

$T_{wall,outside}$ = 105°C

Radiation loss

$$\dot{Q}_{rad} = \varepsilon \cdot C_0 \cdot A \cdot \left[\left(\frac{T_{wall,outside}}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]$$

$$\dot{Q}_{rad} = 0.9 \cdot 5.67 \frac{W}{m^2 \cdot K^4} \cdot 1m^2 \left[\left(\frac{378K}{100} \right)^4 - \left(\frac{293K}{100} \right)^4 \right] = 665.7W$$

Radiation heat transport coefficient

$$\dot{Q}_{rad} = \alpha_{2,rad} \cdot A \cdot \left(T_{wall,outside} - T_2 \right)$$

$$\alpha_{2,rad} = \frac{\dot{Q}_{rad}}{A \cdot \left(T_{walll,outside} - T_2 \right)} = \frac{665.7W}{1m^2 \cdot \left(105°C - 20°C \right)} = 7.83 \frac{W}{m^2 \cdot K}$$

Overall heat transfer coefficient

$$U = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{walll}}{\lambda_{walll}} + \frac{1}{\alpha_{2,conv} + \alpha_{2,rad}}}$$

$$U = \frac{1}{\frac{1}{1300 \frac{W}{m^2 \cdot K}} + \frac{0.002m}{58 \frac{W}{m \cdot K}} + \frac{1}{9 \frac{W}{m^2 \cdot K} + 7.83 \frac{W}{m^2 \cdot K}}} = 16.6 \frac{W}{m^2 \cdot K}$$

Heat loss

$$\dot{Q}_{loss} = U \cdot A \cdot \left(T_1 - T_2 \right) = 16.6 \frac{W}{m^2 \cdot K} \cdot 1m^2 \cdot \left(105°C - 20°C \right) = 1411W$$

Wall outside surface temperature
The same heat power is counted till yhe outer surface only.
Overall heat transport coefficient without convective heat loss and radiation:

$$U^* = \frac{1}{\frac{1}{\alpha_1} + \frac{s_{wall}}{\lambda_{wall}}} = \frac{1}{\frac{1}{1300 \frac{W}{m^2 \cdot K}} + \frac{0.002m}{58 \frac{W}{m \cdot K}}} = 1244 \frac{W}{m^2 \cdot K}$$

$$\dot{Q}_{loss} = U^* \cdot A \cdot \left(T_1 - T'_{wall,outside} \right)$$

$$T'_{wall,outside} = T_1 - \frac{\dot{Q}_{loss}}{U^* \cdot A} = 105°C - \frac{1411W}{1244 \frac{W}{m^2 \cdot K} \cdot 1m^2} = 103.9°C$$

This is almoust the same at the initial estimate, thus the iteration need not be continued.

b) 1 cm wide insulation is put around the oven due to safety reasons. Its heat conductivity is 0.07 W/mK, its emissivity is 0.75. What will be the outside surface temperature?

Initial estimate of wall temperature

Initial estimate of wall temperature can be based on heat resistances.

Total resistance computed in problem a/, together with insulation:

$$R_1 = \frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{w_{ins}}{\lambda_{ins}} + \frac{1}{\alpha_{2,conv} + \alpha_{2,rad}}$$

$$R_1 = \frac{1}{1300\frac{W}{m^2 \cdot K}} + \frac{0.002m}{58\frac{W}{m \cdot K}} + \frac{10^{-2}m}{0.07\frac{W}{m \cdot K}} + \frac{1}{9\frac{W}{m^2 \cdot K} + 7.83\frac{W}{m^2 \cdot K}} = 2.03 \cdot 10^{-1}\frac{m^2 \cdot K}{W}$$

Resistance without convective loss and radiation:

$$R_2 = \frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{w_{ins}}{\lambda_{ins}} = \frac{1}{1300\frac{W}{m^2 \cdot K}} + \frac{0.002m}{58\frac{W}{m \cdot K}} + \frac{10^{-2}m}{0.07\frac{W}{m \cdot K}} = 1.44 \cdot 10^{-1}\frac{m^2 \cdot K}{W}$$

The temperature drops are proportional to the resistances.

$$\frac{T_1 - T_2}{T_1 - T_{wall,outside}} = \frac{R_1}{R_2}$$

$$T_{wall,outside} = T_1 - (T_1 - T_2) \cdot \frac{R_2}{R_1} = 105°C - (105°C - 20°C) \cdot \frac{1.44 \cdot 10^{-1}\frac{m^2 \cdot K}{W}}{2.03 \cdot 10^{-1}\frac{m^2 \cdot K}{W}} = 44.7°C$$

Iteration

Radiation loss

$$\dot{Q}_{rad} = \varepsilon_{ins} \cdot C_0 \cdot A \cdot \left[\left(\frac{T_{wall,outside}}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]$$

$$\dot{Q}_{rad} = 0.75 \cdot 5.67\frac{W}{m^2 \cdot K^4} \cdot 1m^2\left[\left(\frac{317.7K}{100}\right)^4 - \left(\frac{293K}{100}\right)^4\right] = 119.8W$$

Radiation heat transport coefficient

$$\alpha_{2,rad} = \frac{\dot{Q}_{rad}}{A \cdot (T_{wall,outside} - T_2)} = \frac{119.8W}{1m^2 \cdot (44.7°C - 20°C)} = 4.85\frac{W}{m^2 \cdot K}$$

Overall heat transfer coefficient

$$U = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{w_{ins}}{\lambda_{ins}} + \frac{1}{\alpha_{2,vonv} + \alpha_{2,rad}}}$$

$$U = \frac{1}{\frac{1}{1300\frac{W}{m^2\cdot K}} + \frac{0.002m}{58\frac{W}{m\cdot K}} + \frac{10^{-2}m}{0.07\frac{W}{m\cdot K}} + \frac{1}{9\frac{W}{m^2\cdot K} + 4.85\frac{W}{m^2\cdot K}}} = 4.63\frac{W}{m^2\cdot K}$$

Heat loss

$$\dot{Q}_{loss} = U\cdot A\cdot(T_1 - T_2) = 4.63\frac{W}{m^2\cdot K}\cdot 1m^2\cdot(105°C - 20°C) = 393.2W$$

Wall outside surface temperature
The same heat power is counted till the outer surface only.
Overall heat transport coefficient without convective heat loss and radiation:

$$U^* = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{w_{ins}}{\lambda_{ins}}} = \frac{1}{\frac{1}{1300\frac{W}{m^2\cdot K}} + \frac{0.002m}{58\frac{W}{m\cdot K}} + \frac{10^{-2}m}{0.07\frac{W}{m\cdot K}}} = 6.96\frac{W}{m^2\cdot K}$$

$$\dot{Q}_{loss} = U^*\cdot A\cdot(T_1 - T'_{wall,outside})$$

$$T'_{wall,outside} = T_1 - \frac{\dot{Q}_{loss}}{U^*\cdot A} = 105°C - \frac{393.2W}{6.96\frac{W}{m^2\cdot K}\cdot 1m^2} = 48.5°C$$

This is different from the original estimate 44.7°C with several degrees. Iteration is continued.

Radiation loss

$$\dot{Q}'_{rad} = \varepsilon_{ins}\cdot C_0\cdot A\cdot\left[\left(\frac{T_{wall,outside}}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]$$

$$\dot{Q}'_{rad} = 0.75\cdot 5.67\frac{W}{m^2\cdot K^4}\cdot 1m^2\left[\left(\frac{321.5K}{100}\right)^4 - \left(\frac{293K}{100}\right)^4\right] = 140.9W$$

Radiation heat transport coefficient

$$\alpha'_{2,rad} = \frac{\dot{Q}'_{rad}}{A\cdot(T'_{wall,outside} - T_2)} = \frac{140.9W}{1m^2\cdot(48.5°C - 20°C)} = 4.94\frac{W}{m^2\cdot K}$$

Overall heat transfer coefficient

$$U' = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{w_{ins}}{\lambda_{ins}} + \frac{1}{\alpha_{2,vonv} + \alpha'_{2,rad}}}$$

$$U' = \frac{1}{\frac{1}{1300\frac{W}{m^2\cdot K}} + \frac{0.002m}{58\frac{W}{m\cdot K}} + \frac{10^{-2}m}{0.07\frac{W}{m\cdot K}} + \frac{1}{9\frac{W}{m^2\cdot K} + 4.94\frac{W}{m^2\cdot K}}} = 4.64\frac{W}{m^2\cdot K}$$

Heat loss

$$\dot{Q}'_{loss} = U' \cdot A \cdot (T_1 - T_2) = 4.64 \frac{W}{m^2 \cdot K} \cdot 1m^2 \cdot (105°C - 20°C) = 394.6W$$

Wall outside surface temperature

$$\dot{Q}'_{loss} = U^* \cdot A \cdot (T_1 - T''_{wall,outside})$$

$$T''_{wall,outside} = T_1 - \frac{\dot{Q}'_{loss}}{U^* \cdot A} = 105°C - \frac{394.6W}{6.96 \frac{W}{m^2 \cdot K} \cdot 1m^2} = 48.3°C$$

This is almost the same as the earlier value 48.5°C; the iteration is stopped.

$$T''_{wall,outside} = 48.3°C$$

HEAT TRANSPORT IV. Heat transport between wall and fluid (convection)

$$\dot{Q} = \alpha \cdot A \cdot (T_1 - T_2)$$

ahol α film coefficient $\left[\dfrac{W}{m^2 \cdot K}\right]$

Forced flow in tubes

$$Nu = C \cdot Re^a \cdot Pr^b \cdot Vis^c$$

ahol $Nu = \dfrac{D \cdot \alpha}{\lambda}$	Nusselt number	[–]
C	empirical factor	[–]
$Re = \dfrac{D \cdot v \cdot \rho}{\eta}$	Reynolds number	[–]
$Pr = \dfrac{c_p \cdot \eta}{\lambda}$	Prandtl number	[–]
$Vis = \dfrac{\eta_{bulk}}{\eta_{wall}}$	viscosity index	[–]

Material properties are taken at average temperature.

In case of water, aquous solution, and material with neglectable temperature slope of viscosity, $Vis \approx 1$.

water:	$Pr = 3$ to 6	
other liquids:	$Pr > 3$ to 6	(even by decades)
gas:	$Pr \approx 1$	

In mixing process:

$$Re = \frac{d_{paddle}^2 \cdot n \cdot \rho}{\eta}$$

ahol d_{paddle}	diameter of the paddle blade	[m]
n	turning number	[1/s]

$$Nu = \frac{D_{vessel} \cdot \alpha}{\lambda}$$

ahol D_{vessel} internal diameter of the vessel [m]

Problem 8

3 m^3/h glycerol is warmed up with 100°C saturated steam in a double pipe heat exchanger of inner tube 30/36 mm and outer tube 48/54 mm. Glycerol flows in the inner tube, at averaged temperature 75 C, density 1.12 g/cm^3, heat conductivity 0.244 W/mK, specific heat 2410 J/kgK. Heat transfer coefficient at the steam side is 6000 W/m^2K. Heat conductivity of the tube wall is 58 W/mK.
Dynamic viscosity of glycerol in temperature range 65°C to 100°C can ve approximated as

$$\eta_{glycerol} = 2 \cdot 10^6 \,\mathrm{Pas} \cdot e^{-0.05 \frac{1}{K} \cdot T}$$

How much heat power acts over a pipe setcion of 1 m?

Planar wall approximation of the circular wall may be applied.

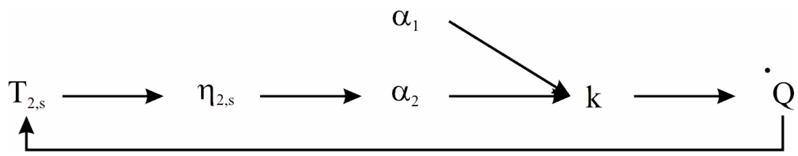
Solution

Summary of data and notation

d_{inner} = 30 mm
d_{outer} = 36 mm
D_{inner} = 48 mm
D_{outer} = 54 mm
L = 1 m
T_1 = 100°C
α_1 = 6000 W/m^2K
λ_{wall} = 58 W/mK

$\dot{V}_2$ = 3 m^3/h
T_2 = 75°C
ρ_2 = 1120 kg/m^3
λ_2 = 0,244 W/mK
$c_{p,2}$ = 2410 J/kgK
$\dot{Q}$ = ?

Solution path



Here k denotes U

Heat transfer coefficient at cold side

Glycerol velocity

$$\dot{V}_2 = A_2 \cdot v_2$$

$$v_2 = \frac{\dot{V}_2}{A_2} = \frac{\dot{V}_2}{\frac{d_{inner}^2 \cdot \pi}{4}} = \frac{3 \frac{m^3}{h}}{\frac{\left(3 \cdot 10^{-2} m\right)^2 \cdot \pi}{4} \cdot 3600 \frac{s}{h}} = 1.18 \frac{m}{s}$$

Glicerol dynamic viscosity at temperature of the bulk

$$\eta_{2,b} = 2 \cdot 10^6 \,\mathrm{Pas} \cdot e^{-0.05 \frac{1}{K} \cdot T_{2,b}} = 2 \cdot 10^6 \,\mathrm{Pas} \cdot e^{-0.05 \frac{1}{K} \cdot 348 K} = 5.55 \cdot 10^{-2} \,\mathrm{Pas}$$

Reynolds number

$$Re_2 = \frac{d_{inner} \cdot v_2 \cdot \rho_2}{\eta_{2,b}} = \frac{3 \cdot 10^{-2} m \cdot 1.18 \frac{m}{s} \cdot 1120 \frac{kg}{m^3}}{5.55 \cdot 10^{-2} \,\mathrm{Pas}} = 714.4$$

According to the charts, Re_2 = 714.4 is in the laminar region.

$$Y_2 = 1.86 \cdot \left(\frac{d_{inner}}{L}\right)^{\frac{1}{3}} \cdot Re_2^{\frac{1}{3}} = 1.86 \cdot \left(\frac{3 \cdot 10^{-2}\,m}{1m}\right)^{\frac{1}{3}} \cdot 714.4^{\frac{1}{3}} = 5.167$$

Prandtl number

$$Pr_2 = \frac{c_{p,2} \cdot \eta_{2,b}}{\lambda_2} = \frac{2410 \frac{J}{kg \cdot K} \cdot 5.55 \cdot 10^{-2}\,Pas}{0.244 \frac{W}{m \cdot K}} = 548.2$$

Nusselt number

$$Y = Nu \cdot \left(\frac{\eta_w}{\eta_b}\right)^{0.14} \cdot Pr^{-\frac{1}{3}}$$

Viscosity index, referring to viscosity at the wall, must be known for computing Nu. Since viscosity at wall is unknown, that must be estimated. Estimation with steam temperature seems a good idea.

$$T_{2,w} = 100°C$$

$$\eta_{2,w} = 2 \cdot 10^6\,Pas \cdot e^{-0.05\frac{1}{K} \cdot T_{2,s}} = 2 \cdot 10^6\,Pas \cdot e^{-0.05\frac{1}{K} \cdot 373K} = 1.59 \cdot 10^{-2}\,Pas$$

$$Nu_2 = Y_2 \cdot \left(\frac{\eta_{2,w}}{\eta_{2,b}}\right)^{-0.14} \cdot Pr_2^{\frac{1}{3}} = 5.167 \cdot \left(\frac{1.59 \cdot 10^{-2}\,Pas}{5.55 \cdot 10^{-2}\,Pas}\right)^{-0.14} \cdot 548.2^{\frac{1}{3}} = 50.38$$

Heat transfer coefficient at cold side

$$Nu = \frac{\alpha \cdot D}{\lambda}$$

$$\alpha_2 = \frac{Nu_2 \cdot \lambda_2}{d_{inner}} = \frac{50.68 \cdot 0.244 \frac{W}{m \cdot K}}{3 \cdot 10^{-2}\,m} = 409.7 \frac{W}{m^2 \cdot K}$$

Heat power

Wall width

$$w_{wall} = \frac{d_{outer} - d_{inner}}{2} = \frac{0.036m - 0.03m}{2} = 0.003m$$

Overall heat transfer coefficient

$$U = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{6000 \frac{W}{m^2 \cdot K}} + \frac{0.003m}{58 \frac{W}{m \cdot K}} + \frac{1}{409.7 \frac{W}{m^2 \cdot K}}} = 376 \frac{W}{m^2 \cdot K}$$

Heat power

$$\dot{Q} = U \cdot A \cdot (T_1 - T_2) = k \cdot \bar{d} \cdot \pi \cdot L \cdot (T_1 - T_2) = U \cdot \frac{d_{inner} + d_{outer}}{2} \cdot \pi \cdot L \cdot (T_1 - T_2)$$

$$\dot{Q} = 376 \frac{W}{m^2 \cdot K} \cdot \frac{0.03m + 0.036m}{2} \cdot \pi \cdot (1m) \cdot (100°C - 75°C) = 974.7W$$

Check of wall temperature

The same heat power but counted till only the inner side of the wall:

$$U^* = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda wall}} = \frac{1}{\frac{1}{6000\frac{W}{m^2 \cdot K}} + \frac{0.003m}{58\frac{W}{m \cdot K}}} = 4579\frac{W}{m^2 \cdot K}$$

$$\dot{Q} = U^* \cdot A \cdot \left(T_1 - T'_{2,w}\right)$$

$$T'_{2,w} = T_1 - \frac{\dot{Q}}{U^* \cdot A} = T_1 - \frac{\dot{Q}}{U^* \cdot \bar{d} \cdot \pi \cdot L} = T_1 - \frac{\dot{Q}}{U^* \cdot \frac{d_{inner} + d_{outer}}{2} \cdot \pi \cdot L}$$

$$T'_{2,w} = 100°C - \frac{974.7W}{4579\frac{W}{m^2 \cdot K} \cdot \frac{0.03m + 0.036m}{2} \cdot \pi \cdot 1m} = 97.9°C$$

This differs with several degrees from the estimate 100°C. However, iteration is based on deviation in viscosity index, because of its small exponent (-0,14).

Viscosity index at 100°C wall temperature:

$$(Vis)^{-0.14} = \left(\frac{\eta_{2,b}}{\eta_{2,w}}\right)^{-0.14} = \left(\frac{5.55 \cdot 10^{-2}\,Pas}{1.59 \cdot 10^{-2}\,Pas}\right)^{-0.14} = 0.839$$

Viscosity index at 97.9°C wall temperature:

$$\eta'_{2,w} = 2 \cdot 10^6\,Pas \cdot e^{-0w05\frac{1}{K} \cdot T'_{2,w}} = 2 \cdot 10^6\,Pas \cdot e^{-0.05\frac{1}{K} \cdot 370.9K} = 1.77 \cdot 10^{-2}\,Pas$$

$$(Vis')^{-0.14} = \left(\frac{\eta_{2,b}}{\eta'_{2,w}}\right)^{-0.14} = \left(\frac{5.55 \cdot 10^{-2}\,Pas}{1.77 \cdot 10^{-2}\,Pas}\right)^{-0.14} = 0.852$$

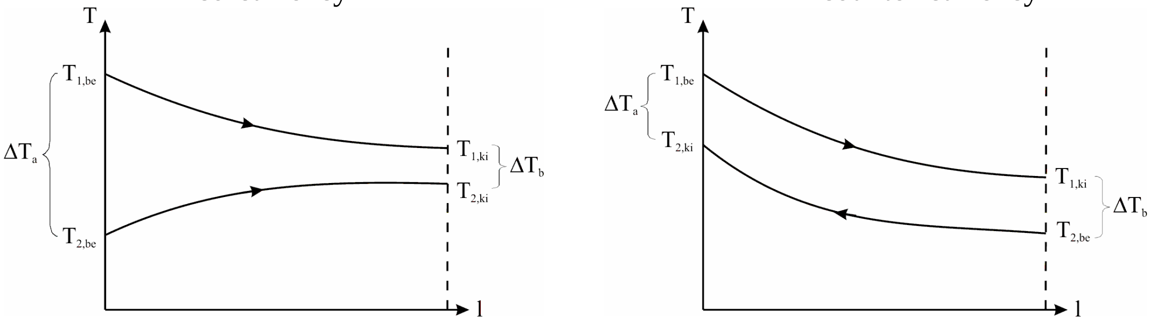
The deviation is about 1.5 % only, acceptable.

$$\dot{Q} = 974{,}7W$$

HEAT TRANSPORT V. Logarithmic approach temperature

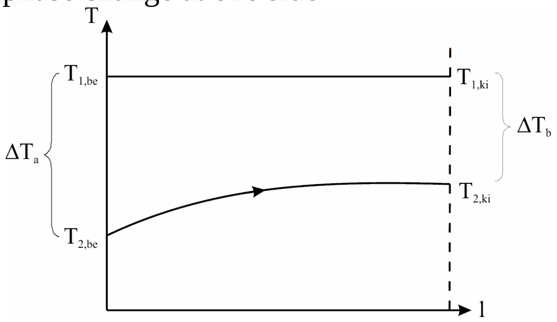
Temperature profiles if temperature changes at both streams

co-currency counter-currency



$$\Delta T_{av} = \frac{\Delta T_a - \Delta T_b}{\ln \dfrac{\Delta T_a}{\Delta T_b}}$$

Temperature profile with phase change at one side



In batch process, the same profiles are formed along time, not the length of the exchanger.

Problem 9

70°C, 1.2 m^3/h ethanol is to be cooled down to 40°C with 1800 kg/h 20°C cooling water in a double pipe heat exchange with inner pipe 16/20 mm, outer pipe 30/35 mm, heat conductivity 58 W/mK. Ethanol flows in the inner pipe, counter-current to the cooling water.

Material data at average temperatures

	ethanol	water
ρ [kg/m^3]	920	994
η [mPas]	1.4	0.656
c_p [kJ/kgK]	3.66	4.18
λ [W/mK]	0.387	0.627

Determine:

a) Outlet temperature of cooling water
b) Average approach temperature
c) Pipe length

Formulas valid to planar walls may be applied.

Solution

a) Outlet temperature of cooling water

Hot stream flow rate

$$\dot{m}_1 = \dot{V}_1 \cdot \rho_{p,1} = 1.2\frac{m^3}{h} \cdot 920\frac{kg}{m^3} = 1104\frac{kg}{h} = 0.307\frac{kg}{s}$$

Heat power

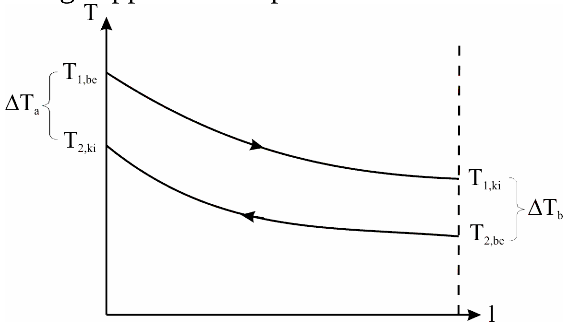
$$\dot{Q} = \dot{m}_1 \cdot c_{p,1} \cdot \left(T_{1,in} - T_{1,out}\right) = 0.307\frac{kg}{s} \cdot 3660\frac{J}{kg \cdot K} \cdot \left(70°C - 40°C\right) = 33672W$$

Cold stream outlet temperature

$$\dot{Q} = \dot{m}_2 \cdot c_{p,2} \cdot \left(T_{2,out} - T_{2,in}\right)$$

$$T_{2,out} = \frac{\dot{Q}}{\dot{m}_2 \cdot c_{p,2}} + T_{2,be} = \frac{33672W \cdot 3600\frac{s}{h}}{1800\frac{kg}{h} \cdot 4180\frac{J}{kg \cdot K}} + 20°C = 36.11°C$$

b) Average approach temperature



Approach temperatures at pipe ends

$$\Delta T_a = T_{1,in} - T_{2,out} = 70°C - 36.11°C = 33.89°C$$

$$\Delta T_b = T_{1,out} - T_{2,in} = 40°C - 20°C = 20°C$$

Logarithmic approach temperature

$$\Delta T_{log} = \frac{\Delta T_a - \Delta T_b}{\ln \frac{\Delta T_a}{\Delta T_b}} = \frac{33.89°C - 20°C}{\ln \frac{33,89°C}{20°C}} = 26.34°C$$

c) Pipe length

Heat transfer coefficient at hot side

Ethanol velocity

$$\dot{V}_1 = A_1 \cdot v_1$$

$$v_1 = \frac{\dot{V}_1}{A_1} = \frac{\dot{V}_1}{\frac{d_{inner}^2 \cdot \pi}{4}} = \frac{1.2 \frac{m^3}{h}}{\frac{(1.6 \cdot 10^{-2} m)^2 \cdot \pi}{4} \cdot 3600 \frac{s}{h}} = 1.66 \frac{m}{s}$$

Reynolds number

$$Re_1 = \frac{d_{inner} \cdot v_1 \cdot \rho_1}{\eta_{1,b}} = \frac{1.6 \cdot 10^{-2} m \cdot 1.66 \frac{m}{s} \cdot 920 \frac{kg}{m^3}}{1.4 \cdot 10^{-3} Pas} = 17431$$

According to charts, Re_1 = 17431 is in turbulent region.

$$Y_1 = 0.023 \cdot Re_1^{0.8} = 0.023 \cdot 17431^{0.8} = 56.86$$

Prandtl number

$$Pr_1 = \frac{c_{p,1} \cdot \eta_1}{\lambda_1} = \frac{3660 \frac{J}{kg \cdot K} \cdot 1.4 \cdot 10^{-3} Pas}{0.387 \frac{W}{m \cdot K}} = 13.24$$

Nusselt number

$$Y = Nu \cdot \left(\frac{\eta_s}{\eta_b}\right)^{0,14} \cdot Pr^{-\frac{1}{3}}$$

$$Nu_1 = Y_1 \cdot (Vis_1)^{-0,14} \cdot Pr_1^{\frac{1}{3}} = 56,86 \cdot 1^{-0,14} \cdot 13,24^{\frac{1}{3}} = 134,5$$

Heat transfer coefficient at hot side

$$Nu = \frac{\alpha \cdot D}{\lambda}$$

$$\alpha_1 = \frac{Nu_1 \cdot \lambda_1}{d_{inner}} = \frac{134.5 \cdot 0.387 \frac{W}{m \cdot K}}{1.6 \cdot 10^{-2} m} = 3254 \frac{W}{m^2 \cdot K}$$

Heat transfer coefficient at cold side

Cooling water velocity

$$\dot{V}_2 = \frac{\dot{m}_2}{\rho_2} = \frac{1800\,\frac{kg}{h}}{994\,\frac{kg}{m^3}\cdot 3600\,\frac{s}{h}} = 5.03\cdot 10^{-4}\,\frac{m^3}{s}$$

$$v_2 = \frac{\dot{V}_2}{A_2} = \frac{\dot{V}_2}{\frac{D_{inner}^2\cdot\pi}{4} - \frac{d_{outer}^2\cdot\pi}{4}} = \frac{5.03\cdot 10^{-4}\,\frac{m^3}{s}}{\frac{(0.03m)^2\cdot\pi}{4} - \frac{(0.02m)^2\cdot\pi}{4}} = 1.28\,\frac{m}{s}$$

Equivalent diameter

$$D_{e,2} = 4\cdot\frac{A_2}{K_2} = 4\cdot\frac{\frac{D_{inner}^2\cdot\pi}{4} - \frac{d_{outer}^2\cdot\pi}{4}}{D_{inner}\cdot\pi + d_{outer}\cdot\pi} = D_{outer} - d_{inner} = 0.03m - 0.02m = 0.01m$$

Reynolds number

$$Re_2 = \frac{D_{e,2}\cdot v_2\cdot\rho_2}{\eta_2} = \frac{0.01m\cdot 1.28\,\frac{m}{s}\cdot 994\,\frac{kg}{m^3}}{0.656\cdot 10^{-3}\,Pas} = 19409$$

According to charts, Re_2 = 19409 is in turbulent region.

$$Y_2 = 0.023\cdot Re_2^{0.8} = 0.023\cdot 19409^{0.8} = 61.96$$

Prandtl number

$$Pr_2 = \frac{c_{p,2}\cdot\eta_2}{\lambda_2} = \frac{4180\,\frac{J}{kg\cdot K}\cdot 0.656\cdot 10^{-3}\,Pas}{0.627\,\frac{W}{m\cdot K}} = 4.37$$

Nusselt number

$$Nu_2 = Y_2\cdot\left(\frac{\eta_{2,w}}{\eta_{2,b}}\right)^{-0.14}\cdot Pr_2^{\frac{1}{3}} = 61.96\cdot 1^{-0.14}\cdot 4.37^{\frac{1}{3}} = 101.3$$

Heat transfer coefficient at cold side

$$\alpha_2 = \frac{Nu_2\cdot\lambda_2}{D_{e,2}} = \frac{101.3\cdot 0.627\,\frac{W}{m\cdot K}}{0.01m} = 6352\,\frac{W}{m^2\cdot K}$$

Overall heat transfer coefficient

Wall thickness

$$w_{wall} = \frac{d_{outer} - d_{inner}}{2} = \frac{0.016m - 0.02m}{2} = 0.002m$$

Overall heat transfer coefficient

$$U = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{3254\,\frac{W}{m^2\cdot K}} + \frac{0.002m}{58\,\frac{W}{m\cdot K}} + \frac{1}{6352\,\frac{W}{m^2\cdot K}}} = 2003\,\frac{W}{m^2\cdot K}$$

Exchanger length

Heat transfer area

$$\dot{Q} = k \cdot A \cdot \Delta T_{av}$$

$$A = \frac{\dot{Q}}{k \cdot \Delta T_{av}} = \frac{33672\text{W}}{2003\frac{\text{W}}{\text{m}^2 \cdot \text{K}} \cdot 26.34°\text{C}} = 0.638\text{m}^2$$

Length

$$A = L \cdot \bar{d} \cdot \pi = L \cdot \frac{d_{inner} + d_{outer}}{2} \cdot \pi$$

$$L = \frac{A}{\frac{d_{inner} + d_{outer}}{2} \cdot \pi} = \frac{0.638\text{m}^2}{\frac{0.016\text{m} + 0.02\text{m}}{2} \cdot \pi} = 11.3\text{m}$$

Problem 10

Inner diameter of the inner tube of a double pipe heat exchanger is 30 mm. 50 % aquous glycerol solution flows in that tube with velocity 1.07 m/s, and cools down from 80 C to 60 C. Material properties at average temperature are $\eta = 1.8\cdot10^{-3}$ Pas, $\lambda = 0.285$ W/mK, $c_p = 3.39$ kJ/kgK, $\rho = 1120$ kg/m^3.
The wall of the inner tube is 2 mm thick, its heat conductivity is $\lambda = 62.8$ W/mK.
Cooling water of 20 C enters between the two tubes and flows with velocity 0.8 m/s. Its properties at average temperature are $\eta = 10^{-3}$ Pas, $\lambda = 0.628$ W/mK, $c_p = 4.18$ kJ/kgK, $\rho = 1000$ kg/m^3.
Internal diameter of the outer tube is 48.8 mm.

How long the exchanger should be if the cooling water flows a/ co-current, b/ counter-current, to the glycerol solution?

Formulas valid to planar walls may be applied.

Solution

Data and notation

$T_{1,in} = 80°C$
$T_{1,out} = 60°C$
$v_1 = 1.07$ m/s
$\eta_1 = 1.8\cdot10^{-3}$ Pas
$\lambda_1 = 0.285$ W/mK
$c_{p,1} = 3.39$ kJ/kgK
$\rho_1 = 1120$ kg/m^3
$T_{2,in} = 20°C$
$v_2 = 0.8$ m/s

$\eta_2 = 10^{-3}$ Pas
$\lambda_2 = 0.628$ W/mK
$c_{p,2} = 4.18$ kJ/kgK
$\rho_2 = 1000$ kg/m^3
$d_{inner} = 30$ mm
$w_{wall} = 2$ mm
$D_{inner} = 48.8$ mm
$\lambda_{wall} = 62.8$ W/mK

Heat transfer coefficient at hot side

Reynolds number

$$\mathrm{Re}_1 = \frac{d_{inner}\cdot v_1\cdot\rho_1}{\eta_{1,b}} = \frac{3\cdot10^{-2}\,\mathrm{m}\cdot1.07\,\frac{\mathrm{m}}{\mathrm{s}}\cdot1120\,\frac{\mathrm{kg}}{\mathrm{m}^3}}{1.8\cdot10^{-3}\,\mathrm{Pas}} = 2\cdot10^4$$

$\mathrm{Re}_1 = 2\cdot10^4$ is in turbulent region.

$$Y_1 = 0.023\cdot\mathrm{Re}_1^{0.8} = 0{,}023\cdot\left(2\cdot10^4\right)^{0.8} = 63.4$$

Prandtl number

$$\mathrm{Pr}_1 = \frac{c_{p,1}\cdot\eta_1}{\lambda_1} = \frac{3390\,\frac{\mathrm{J}}{\mathrm{kg\cdot K}}\cdot1.8\cdot10^{-3}\,\mathrm{Pas}}{0.285\,\frac{\mathrm{W}}{\mathrm{m\cdot K}}} = 21.41$$

Nusselt number

$$Y = Nu \cdot \left(\frac{\eta_w}{\eta_b} \right)^{0.14} \cdot Pr^{-\frac{1}{3}}$$

$$Nu_1 = Y_1 \cdot (Vis_1)^{-0.14} \cdot Pr_1^{\frac{1}{3}} = 63.4 \cdot 1^{-0.14} \cdot 21.41^{\frac{1}{3}} = 176$$

Heat transfer coefficient at hot side

$$Nu = \frac{\alpha \cdot D}{\lambda}$$

$$\alpha_1 = \frac{Nu_1 \cdot \lambda_1}{d_{inner}} = \frac{176 \cdot 0.285 \frac{W}{m \cdot K}}{3 \cdot 10^{-2} m} = 1672.44 \frac{W}{m^2 \cdot K}$$

Heat transfer coefficient at cold side

Equivalent diameter

$$D_{e,2} = 4 \cdot \frac{A_2}{C_2} = 4 \cdot \frac{\frac{D_{inner}^2 \cdot \pi}{4} - \frac{d_{outer}^2 \cdot \pi}{4}}{D_{inner} \cdot \pi + d_{outer} \cdot \pi} = D_{inner} - d_{outer} = D_{inner} - (d_{inner} + 2 \cdot w_{wall})$$

$$D_{e,2} = 48.8mm - (30mm + 2 \cdot 2mm) = 14.8mm$$

Reynolds number

$$Re_2 = \frac{D_{e,2} \cdot v_2 \cdot \rho_2}{\eta_2} = \frac{1.48 \cdot 10^{-2} m \cdot 0.8 \frac{m}{s} \cdot 1000 \frac{kg}{m^3}}{10^{-3} Pas} = 11840$$

Re_2 = 11840 is in turbulent region.

$$Y_2 = 0.023 \cdot Re_2^{0.8} = 0.023 \cdot 11840^{0.8} = 41.73$$

Prandtl number

$$Pr_2 = \frac{c_{p,2} \cdot \eta_2}{\lambda_2} = \frac{4180 \frac{J}{kg \cdot K} \cdot 10^{-3} Pas}{0.628 \frac{W}{m \cdot K}} = 6.66$$

Nusselt number

$$Nu_2 = Y_2 \cdot \left(\frac{\eta_{2,w}}{\eta_{2,b}} \right)^{-0.14} \cdot Pr_2^{\frac{1}{3}} = 41.73 \cdot 1^{-0.14} \cdot 6.66^{\frac{1}{3}} = 78.49$$

Heat transfer coefficient at cold side

$$\alpha_2 = \frac{Nu_2 \cdot \lambda_2}{D_{e,2}} = \frac{78.49 \cdot 0.628 \frac{W}{m \cdot K}}{1.48 \cdot 10^{-2} m} = 3330.5 \frac{W}{m^2 \cdot K}$$

Overall heat transfer coefficient

$$U = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{1672.44 \frac{W}{m^2 \cdot K}} + \frac{0.002m}{62.8 \frac{W}{m \cdot K}} + \frac{1}{3330.5 \frac{W}{m^2 \cdot K}}} = 1075.23 \frac{W}{m^2 \cdot K}$$

Cooling water's outlet temperature: based on heat balance

Hot stream volumetric flow rate

$$\dot{V}_1 = v_1 \cdot A_1 = v_1 \cdot \frac{d_{inner}^2 \cdot \pi}{4} = 1.07 \frac{m}{s} \cdot \frac{(0.03m)^2 \cdot \pi}{4} = 7.56 \cdot 10^{-4} \frac{m^3}{s}$$

Hot stream mass flow rate

$$\dot{m}_1 = \dot{V}_1 \cdot \rho_{p,1} = 7.56 \cdot 10^{-4} \frac{m^3}{s} \cdot 1120 \frac{kg}{m^3} = 0.847 \frac{kg}{s}$$

Heat power

$$\dot{Q} = \dot{m}_1 \cdot c_{p,1} \cdot (T_{1,in} - T_{1,out}) = 0.847 \frac{kg}{s} \cdot 3390 \frac{J}{kg \cdot K} \cdot (80°C - 60°C) = 5.74 \cdot 10^4 W$$

Cold stream volumetric flow rate

$$\dot{V}_2 = v_2 \cdot A_2 = v_2 \cdot \left(\frac{D_{inner}^2 \cdot \pi}{4} - \frac{d_{outer}^2 \cdot \pi}{4} \right) = v_2 \cdot \left(\frac{D_{inner}^2 \cdot \pi}{4} - \frac{(d_{inner} + 2 \cdot w_{wall})^2 \cdot \pi}{4} \right)$$

$$\dot{V}_2 = 0.8 \frac{m}{s} \cdot \left(\frac{(4.88 \cdot 10^{-2} m)^2 \cdot \pi}{4} - \frac{(3 \cdot 10^{-2} m + 2 \cdot 2 \cdot 10^{-3} m)^2 \cdot \pi}{4} \right) = 7.7 \cdot 10^{-4} \frac{m^3}{s}$$

Cold stream mass flow rate

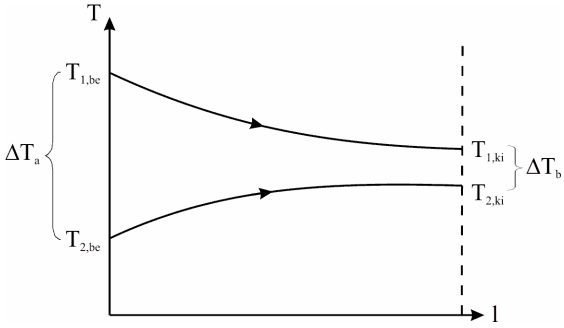
$$\dot{m}_2 = \dot{V}_2 \cdot \rho_{p,2} = 7.7 \cdot 10^{-4} \frac{m^3}{s} \cdot 1000 \frac{kg}{m^3} = 0.77 \frac{kg}{s}$$

Cold stream outlet temperature

$$\dot{Q} = \dot{m}_2 \cdot c_{p,2} \cdot (T_{2,out} - T_{2,in})$$

$$T_{2,out} = T_{2,in} + \frac{\dot{Q}}{\dot{m}_2 \cdot c_{p,2}} = 20°C + \frac{5.74 \cdot 10^4 W}{0.77 \frac{kg}{s} \cdot 4180 \frac{J}{kg \cdot K}} = 37.84°C$$

a/ Co-currency



Approach temperatures at the two endpoints.

$$\Delta T_a = T_{1,in} - T_{2,in} = 80°C - 20°C = 60°C$$

$$\Delta T_b = T_{1,out} - T_{2,out} = 60°C - 37.84°C = 22.16°C$$

Logarithmic approach temperature

$$\Delta T_{av} = \frac{\Delta T_a - \Delta T_b}{\ln \frac{\Delta T_a}{\Delta T_b}} = \frac{60°C - 22{,}16°C}{\ln \frac{60°C}{22.16°C}} = 37.99°C$$

Heat transfer area

$$\dot{Q} = U \cdot A \cdot \Delta T_{av}$$

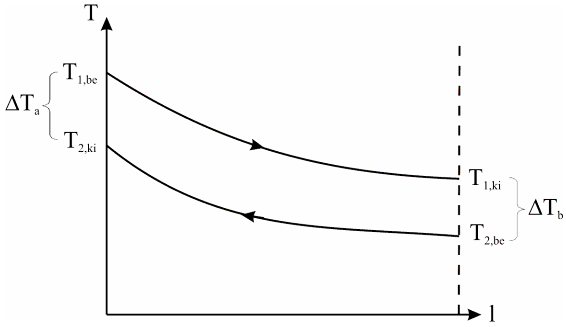
$$A = \frac{\dot{Q}}{U \cdot \Delta T_{av}} = \frac{5.74 \cdot 10^4\,W}{1075.23 \frac{W}{m^2 \cdot K} \cdot 37.99°C} = 1.41m^2$$

Length

$$A = L \cdot \bar{d} \cdot \pi = L \cdot \frac{d_{inner} + d_{outer}}{2} \cdot \pi = L \cdot \left(d_{inner} + w_{wall}\right) \cdot \pi$$

$$L = \frac{A}{\left(d_{inner} + w_{wall}\right) \cdot \pi} = \frac{1.41m^2}{\left(0.03m + 0.002m\right) \cdot \pi} = 14m$$

a/ Counter-currency



Approach temperatures at the two endpoints.

$$\Delta T_a = T_{1,be} - T_{2,ki} = 80°C - 37,84°C = 42,16°C$$

$$\Delta T_b = T_{1,ki} - T_{2,be} = 60°C - 20°C = 40°C$$

Logarithmic approach temperature

$$\Delta T_{av} = \frac{\Delta T_a - \Delta T_b}{\ln \frac{\Delta T_a}{\Delta T_b}} = \frac{42.16°C - 40°C}{\ln \frac{42.16°C}{40°C}} = 41.07°C$$

Heat transfer area

$$A = \frac{\dot{Q}}{U \cdot \Delta T_{av}} = \frac{5.74 \cdot 10^4 \, W}{1075.23 \frac{W}{m^2 \cdot K} \cdot 41.07°C} = 1.3 m^2$$

Length

$$L = \frac{A}{(d_{inner} + w_{wall}) \cdot \pi} = \frac{1.3 m^2}{(0.03 m + 0.002 m) \cdot \pi} = 12.9 m$$

6.11. feladat

1.8 m^3 sodium hydroxide solution is warmed up from 40 C to 140 C with saturated steam in a jacketed and mixed vessel of diameter 1.2 m. A paddle of 300 mm is applied with turning rate 120 1/min. The heat transfer surface area is 7.2 m^2, inner wall thickness is 10 mm, heat conductivity of the wall is 58 W/mK. Heat transfer coefficient at the steam side is 6500 W/m^2K.
Material properties of the NaOH solution at average temperature are as follow. Density: 1.43 g/cm^3, viscosity: 0.65 mPas, heat conductivity: 0.588 W/mK, specific heat: 3137 J/kgK.

How long does warming up the NaOH solution last?

Formulas valid to planar walls may be applied.

Solution

Notation

$V_2 = 1,8\ m^3$
$T_{2,in} = 40°C$
$T_{2,out} = 120°C$
$T_1 = 140°C$
$D=D_{vessel} = 1.2\ m$
$d=d_{paddle} = 300\ mm = 0.3\ m$
$n = 120\ 1/min = 2\ 1/s$
$A = 7.2\ m^2$

$s_{fal} = 10\ mm = 0.01\ m$
$\lambda_{fal} = 58\ W/mK$
$\alpha_1 = 6500\ W/m^2K$
$\rho_2 = 1.43\ g/cm^3 = 1430\ kg/m^3$
$\eta_2 = 0.65\ mPas = 0.65\cdot10^{-3}\ Pas$
$\lambda_2 = 0.588\ W/mK$
$c_{p,2} = 3137\ J/kgK$

$t = ?$

Heat transfer coefficient at cold side

Reynolds number

$$Re_2 = \frac{d^2 \cdot n \cdot \rho_2}{\eta_2} = \frac{(0.3\,m)^2 \cdot 2\frac{1}{s} \cdot 1430\frac{kg}{m^3}}{0.65\cdot10^{-3}\,Pas} = 3.96\cdot10^5$$

Chart related to mixed vessel

$$Y_2 = 0.37\cdot Re_2^{\frac{2}{3}} = 0.37\cdot\left(3.96\cdot10^5\right)^{\frac{2}{3}} = 1995$$

Prandtl number

$$Pr_2 = \frac{c_{p,2}\cdot\eta_2}{\lambda_2} = \frac{3137\frac{J}{kg\cdot K}\cdot0.65\cdot10^{-3}\,Pas}{0.588\frac{W}{m\cdot K}} = 3.47$$

Nusselt number

$$Nu_2 = Y_2\cdot Pr_2^{\frac{1}{3}} = 1995\cdot3.47^{\frac{1}{3}} = 3021$$

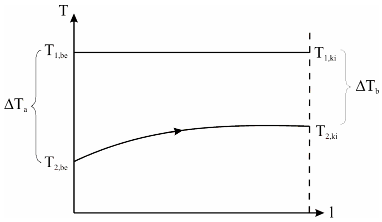
Heat transfer coefficient at cold side

$$\alpha_2 = \frac{Nu_2 \cdot \lambda_2}{D} = \frac{3021 \cdot 0.588 \frac{W}{m \cdot K}}{1.2m} = 1480 \frac{W}{m^2 \cdot K}$$

Overall heat transfer coefficient

$$U = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{6500 \frac{W}{m^2 \cdot K}} + \frac{0.01m}{58 \frac{W}{m \cdot K}} + \frac{1}{1480 \frac{W}{m^2 \cdot K}}} = 998 \frac{W}{m^2 \cdot K}$$

Logarithmic approach temperature



Initial and final approach temperatures

$$\Delta T_a = T_1 - T_{2,in} = 140°C - 40°C = 100°C$$

$$\Delta T_b = T_1 - T_{2,out} = 140°C - 120°C = 20°C$$

Logarithmic approach temperature

$$\Delta T_{av} = \frac{\Delta T_a - \Delta T_b}{\ln \frac{\Delta T_a}{\Delta T_b}} = \frac{100°C - 20°C}{\ln \frac{100°C}{20°C}} = 49.7°C$$

Átment hőáram

$$\dot{Q} = k \cdot A \cdot \Delta T_{átl} = 998 \frac{W}{m^2 \cdot K} \cdot 7{,}2m^2 \cdot 49{,}7°C = 3{,}57 \cdot 10^5 W$$

Heat needed for warming

Formula $\dot{Q} = \dot{m}_2 \cdot c_{p,2} \cdot (T_{2,out} - T_{2,in})$ is meaningless (a conceptual error) here because there is no mass flow rate of NaOH solution. This a batch process on NaOH side.

$$Q = m_2 \cdot c_{p,2} \cdot (T_{2,out} - T_{2,in}) = V_2 \cdot \rho_2 \cdot c_{p,2} \cdot (T_{2,out} - T_{2,in})$$

$$Q = 1.8m^3 \cdot 1430 \frac{kg}{m^3} \cdot 3137 \frac{J}{kg \cdot K} \cdot (120°C - 40°C) = 6.46 \cdot 10^8 J$$

Time of warming up

$$t = \frac{Q}{\dot{Q}} = \frac{6.46 \cdot 10^8 J}{3.57 \cdot 10^5 W} = 1809 s = 30.16 min$$

HEAT TRANSPORT VI. Shell-and-tube heat exchangers

Problem 12

250 m^3/h iso-propanol has to be cooled down from 82.5°C to 50°C, with cooling water at 20°C, but it is available up to most 600 m^3/h only. There is a one-pass shell-and-tube heat exchanger for this aim, with 91 tubes of diameter 25/30 mm and heat conductivity 58 W/mK. Inner diameter of the shell is 45 cm. The length of the exchanger is 1.4 m. The exchanger is used in co-current way, with cooling water flowing in the tubes.
Is this heat exchanger applicable for the task?

Material data at average temperatures

	iso-propanol	water
ρ [kg/m^3]	770	994
η [mPas]	0.85	0.656
c_p [J/kgK]	3054	4180
λ [W/mK]	0.156	0.627

Formulas valid to planar walls may be applied.

Solution

Notation

$\dot{V}_1 = 250\ m^3/h$
$T_{1,in} = 82{,}5°C$
$T_{1,out} = 50°C$
$\dot{V}_2 = 600\ m^3/h$
$T_{2,in} = 20°C$

$N = 91$
$d_{inner} = 25\ mm$
$d_{outer} = 30\ mm$
$D_{inner} = 45\ cm$
$L = 1{,}4\ m$
$\lambda_{wall} = 58\ W/mK$

Heat transfer coefficient at hot side

Iso-propanol velocity

$$\dot{V}_1 = A_1 \cdot v_1$$

$$v_1 = \frac{\dot{V}_1}{A_1} = \frac{\dot{V}_1}{N \cdot \frac{d_{inner}^2 \cdot \pi}{4}} = \frac{250\,\frac{m^3}{h}}{91 \cdot \frac{(2.5 \cdot 10^{-2}\,m)^2 \cdot \pi}{4} \cdot 3600\,\frac{s}{h}} = 1.55\,\frac{m}{s}$$

Reynolds number

$$Re_1 = \frac{d_{inner} \cdot v_1 \cdot \rho_1}{\eta_1} = \frac{2.5 \cdot 10^{-2}\,m \cdot 1.55\,\frac{m}{s} \cdot 770\,\frac{kg}{m^3}}{0.85 \cdot 10^{-3}\,Pas} = 3.52 \cdot 10^4$$

$Re_1 = 3{,}52 \cdot 10^4$ is in turbulent region.

$$Y_1 = 0.023 \cdot Re_1^{0.8} = 0.023 \cdot \left(3.52 \cdot 10^4\right)^{0.8} = 99.78$$

Prandtl number

$$Pr_1 = \frac{c_{p,1} \cdot \eta_1}{\lambda_1} = \frac{3054 \frac{J}{kg \cdot K} \cdot 0.85 \cdot 10^{-3} Pas}{0.156 \frac{W}{m \cdot K}} = 16.64$$

Nusselt number

$$Y = Nu \cdot \left(\frac{\eta_w}{\eta_b} \right)^{0.14} \cdot Pr^{-\frac{1}{3}}$$

$$Nu_1 = Y_1 \cdot (Vis_1)^{-0.14} \cdot Pr_1^{\frac{1}{3}} = 99.78 \cdot 1^{-0.14} \cdot 16.64^{\frac{1}{3}} = 254.7$$

Heat transfer coefficient at hot side

$$Nu = \frac{\alpha \cdot D}{\lambda}$$

$$\alpha_1 = \frac{Nu_1 \cdot \lambda_1}{d_{inner}} = \frac{254.7 \cdot 0.156 \frac{W}{m \cdot K}}{2.5 \cdot 10^{-2} m} = 1590 \frac{W}{m^2 \cdot K}$$

Heat transfer coefficient at cold side

Equivalent diameter

$$D_{e,2} = 4 \cdot \frac{A_2}{K_2} = 4 \cdot \frac{\frac{D_{inner}^2 \cdot \pi}{4} - N \cdot \frac{d_{inner}^2 \cdot \pi}{4}}{D_{inner} \cdot \pi + N \cdot d_{outer} \cdot \pi} = 4 \cdot \frac{\frac{(0.45m)^2 \cdot \pi}{4} - 91 \cdot \frac{(0.03m)^2 \cdot \pi}{4}}{0.45m \cdot \pi + 91 \cdot 0.03m \cdot \pi} = 3.8 \cdot 10^{-2} m$$

Cooling water velocity

$$\dot{V}_2 = A_2 \cdot v_2$$

$$v_2 = \frac{\dot{V}_2}{A_2} = \frac{\dot{V}_2}{\frac{D_{inner}^2 \cdot \pi}{4} - N \cdot \frac{d_{outer}^2 \cdot \pi}{4}} = \frac{600 \frac{m^3}{h}}{\left[\frac{(0.45m)^2 \cdot \pi}{4} - 91 \cdot \frac{(0.03m)^2 \cdot \pi}{4} \right] \cdot 3600 \frac{s}{h}} = 1.76 \frac{m}{s}$$

Reynolds number

$$Re_2 = \frac{D_{e,2} \cdot v_2 \cdot \rho_2}{\eta_2} = \frac{3.8 \cdot 10^{-2} m \cdot 1.76 \frac{m}{s} \cdot 994 \frac{kg}{m^3}}{0.656 \cdot 10^{-3} Pas} = 1.01 \cdot 10^5$$

$Re_2 = 1{,}01 \cdot 10^5$ is in turbulent region.

$$Y_2 = 0.023 \cdot Re_2^{0.8} = 0.023 \cdot \left(1.01 \cdot 10^5\right)^{0.8} = 232.5$$

Prandtl number

$$Pr_2 = \frac{c_{p,2} \cdot \eta_2}{\lambda_2} = \frac{4180 \frac{J}{kg \cdot K} \cdot 0.656 \cdot 10^{-3} Pas}{0.627 \frac{W}{m \cdot K}} = 4.37$$

Nusselt number

$$Nu_2 = Y_2 \cdot \left(\frac{\eta_{2,w}}{\eta_{2,b}}\right)^{-0.14} \cdot Pr_2^{\frac{1}{3}} = 232.5 \cdot 1^{-0.14} \cdot 4.37^{\frac{1}{3}} = 380$$

Heat transfer coefficient at cold side

$$\alpha_2 = \frac{Nu_2 \cdot \lambda_2}{D_{e,2}} = \frac{380 \cdot 0.627 \frac{W}{m \cdot K}}{3.8 \cdot 10^{-2} m} = 6271 \frac{W}{m^2 \cdot K}$$

Overall heat transfer coefficient

Wall thickness

$$w_w = \frac{d_{outer} - d_{inner}}{2} = \frac{0.03m - 0.025m}{2} = 0.0025m$$

Overall heat transfer coefficient

$$U = \frac{1}{\frac{1}{\alpha_1} + \frac{w_{wall}}{\lambda_{wall}} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{1590 \frac{W}{m^2 \cdot K}} + \frac{0.0025m}{58 \frac{W}{m \cdot K}} + \frac{1}{6271 \frac{W}{m^2 \cdot K}}} = 1203 \frac{W}{m^2 \cdot K}$$

Cold stream outlet temperature (from heat balance)

Hot stream mass flow rate

$$\dot{m}_1 = \dot{V}_1 \cdot \rho_{p,1} = 250 \frac{m^3}{h} \cdot 770 \frac{kg}{m^3} = 192500 \frac{kg}{h} = 53.47 \frac{kg}{s}$$

Heat power

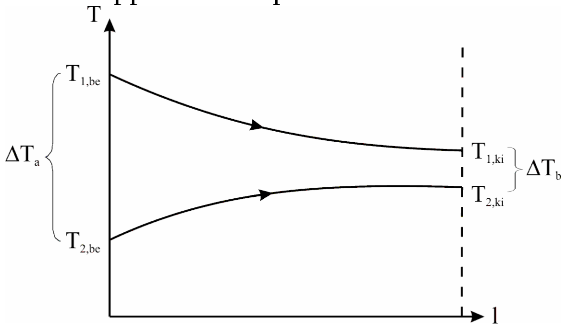
$$\dot{Q} = \dot{m}_1 \cdot c_{p,1} \cdot \left(T_{1,in} - T_{1,out}\right) = 53.47 \frac{kg}{s} \cdot 3054 \frac{J}{kg \cdot K} \cdot \left(82.5°C - 50°C\right) = 5.31 \cdot 10^6 W$$

Cold stream outlet temperature

$$\dot{Q} = \dot{m}_2 \cdot c_{p,2} \cdot \left(T_{2,out} - T_{2,in}\right)$$

$$T_{2,out} = \frac{\dot{Q}}{\dot{m}_2 \cdot c_{p,2}} + T_{2,in} = \frac{\dot{Q}}{\dot{V}_2 \cdot \rho_2 \cdot c_{p,2}} + T_{2,in} = \frac{5.31 \cdot 10^6 W \cdot 3600 \frac{s}{h}}{600 \frac{m^3}{h} \cdot 994 \frac{kg}{m^3} 4180 \frac{J}{kg \cdot K}} + 20°C = 27.66°C$$

Logarithmic approach temperature



Approach temperatures at the two ends

$$\Delta T_a = T_{1,in} - T_{2,in} = 82.5°C - 20°C = 62.5°C$$

$$\Delta T_b = T_{1,out} - T_{2,out} = 50°C - 27.66°C = 22.34°C$$

Logarithmic approach temperature

$$\Delta T_{av} = \frac{\Delta T_a - \Delta T_b}{\ln \frac{\Delta T_a}{\Delta T_b}} = \frac{62.5°C - 22.34°C}{\ln \frac{62.5°C}{22.34°C}} = 39.04°C$$

Comparison of heat transfer areas

Needed area

$$\dot{Q} = U \cdot A \cdot \Delta T_{av}$$

$$A = \frac{\dot{Q}}{U \cdot \Delta T_{av}} = \frac{5.31 \cdot 10^6 W}{1203 \frac{W}{m^2 \cdot K} \cdot 39.04°C} = 113m^2$$

Available area

$$A' = N \cdot \frac{d_{inner} + d_{outer}}{2} \cdot \pi \cdot L = 91 \cdot \frac{0.025m + 0.03m}{2} \cdot \pi \cdot 1.4 = 11m^2$$

Area of the available heat exchanger is approximately one tenth of what is needed, thus it is not applicable for this aim.

Problem 13

6 t/h 80°C wet steam of qualty 0.4 kg water/kg steam is to be condensed in a shell-and-tube condenser containing 37 tubes of diameter 30/40 mm. How long should the tubes be if the cooling water warms up in the tubes from 20 C to 30 C, and the steam side heat transfer coefficient is 5815 W/m^{2}K? Thermal resistance of the wall can be neglected.

Solution

Notation

$x = 0.6$ (steam quality)
$\dot{m}_1 = 6000$ kg/h
$T_1 = 80°C$
$N = 37$
$d_{inner} = 30$ mm
$d_{outer} = 40$ mm
$T_{2,in} = 20°C$
$T_{2,out} = 30°C$
$\alpha_1 = 5815$ W/m^{2}K

Cooling water velocity (from heat balance)

Net steam mass flow rate

$$\dot{m}_{st} = x \cdot \dot{m}_1 = \frac{0.6 \cdot 6000 \frac{kg}{h}}{3600 \frac{s}{h}} = 1 \frac{kg}{s}$$

Vaporization heat from steam table (at 80 C):

$$\Delta H^{vap} = 2308.183 \frac{kJ}{kg}$$

Heat power

$$\dot{Q} = \dot{m}_{st} \cdot \Delta H^{vap} = 1 \frac{kg}{s} \cdot 2308.183 \frac{kJ}{kg} = 2308.183\,kW$$

Cold stream mass flow rate

$$\dot{Q} = \dot{m}_2 \cdot c_{p,2} \cdot \left(T_{2,out} - T_{2,in}\right)$$

$$\dot{m}_2 = \frac{\dot{Q}}{c_{p,2} \cdot \left(T_{2,out} - T_{2,in}\right)} = \frac{2308.183\,kW}{4.18 \frac{kJ}{kg \cdot K} \cdot \left(30°C - 20°C\right)} = 55.22 \frac{kg}{s}$$

Cold stream volumetric flow rate

$$\dot{V}_2 = \frac{\dot{m}_2}{\rho_2} = \frac{55.22\,\frac{kg}{s}}{1000\,\frac{kg}{m^3}} = 5.52\cdot 10^{-2}\,\frac{m^3}{s}$$

Cold stream velocity

$$v_2 = \frac{\dot{V}_2}{A_2} = \frac{\dot{V}_2}{N\cdot\frac{d_{inner}^2\cdot\pi}{4}} = \frac{5.52\cdot 10^{-2}\,\frac{m^3}{s}}{37\cdot\frac{(0.03m)^2\cdot\pi}{4}} = 2.11\,\frac{m}{s}$$

Heat transfer coefficient at cold side

Reynolds number

$$Re_2 = \frac{d_{iner}\cdot v_2\cdot\rho_2}{\eta_2} = \frac{0.03\,m\cdot 2.11\,\frac{m}{s}\cdot 1000\,\frac{kg}{m^3}}{10^{-3}\,Pas} = 6.33\cdot 10^4$$

$Re_2 = 6{,}33\cdot 10^4$ is in turbulent region.

$$Y_2 = 0.023\cdot Re_2^{0.8} = 0.023\cdot\left(6.33\cdot 10^4\right)^{0.8} = 159.44$$

Prandtl number

$$Pr_2 = \frac{c_{p,2}\cdot\eta_2}{\lambda_2} = \frac{4180\,\frac{J}{kg\cdot K}\cdot 10^{-3}\,Pas}{0.628\,\frac{W}{m\cdot K}} = 6.66$$

Nusselt number

$$Nu_2 = Y_2\cdot\left(\frac{\eta_{2,w}}{\eta_{2,b}}\right).\cdot Pr_2^{\frac{1}{3}} = 159.44\cdot 1^{-0.14}\cdot 6.66^{\frac{1}{3}} = 300$$

Heat transfer coefficient at cold side

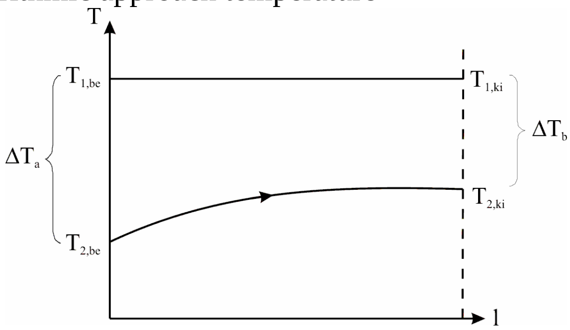
$$\alpha_2 = \frac{Nu_2\cdot\lambda_2}{d_{inner}} = \frac{300\cdot 0.628\,\frac{W}{m\cdot K}}{0.03m} = 6280\,\frac{W}{m^2\cdot K}$$

Overall heat transfer coefficient

(Wall resistance is neglected)

$$U = \frac{1}{\frac{1}{\alpha_1}+\frac{1}{\alpha_2}} = \frac{1}{\frac{1}{5815\,\frac{W}{m^2\cdot K}}+\frac{1}{6280\,\frac{W}{m^2\cdot K}}} = 3019\,\frac{W}{m^2\cdot K}$$

Logarithmic approach temperature



Approach temperatures at the two endpoints.

$$\Delta T_a = T_1 - T_{2,in} = 80°C - 20°C = 60°C$$

$$\Delta T_b = T_1 - T_{2,out} = 80°C - 30°C = 50°C$$

Logarithmic approach temperature

$$\Delta T_{av} = \frac{\Delta T_a - \Delta T_b}{\ln \frac{\Delta T_a}{\Delta T_b}} = \frac{60°C - 50°C}{\ln \frac{60°C}{50°C}} = 54.85°C$$

Heat transfer area

$$\dot{Q} = U \cdot A \cdot \Delta T_{av}$$

$$A = \frac{\dot{Q}}{U \cdot \Delta T_{av}} = \frac{2\,308183\,W}{3019 \frac{W}{m^2 \cdot K} \cdot 54.85°C} = 13.94 m^2$$

Needed length

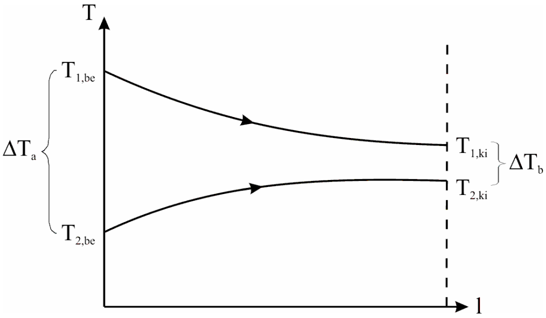
$$A = N \cdot \frac{d_{inner} + d_{outer}}{2} \cdot \pi \cdot L$$

$$L = \frac{A}{N \cdot \frac{d_{inner} + d_{outer}}{2} \cdot \pi} = \frac{13.94 m^2}{37 \cdot \frac{0.03m + 0.04m}{2} \cdot \pi} = 3.42m$$

HEAT TRANSPORT VII. Calculation of outlet temperatures

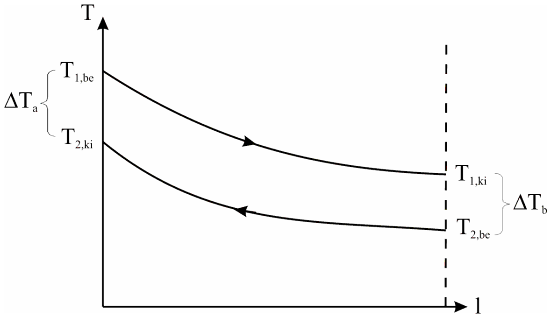
This is needed if both stream temperatures change and only the inlet temperatures are known for both streams.

Co-current



The whole temperature profile an be calculated

Counter-current



Only the end temperatures can be calculated

Heat capacity rates

=Heat power needed to change the temperature of the stream by 1 degree.

$$q_{w,1} = \dot{m}_1 \cdot c_{p,1}$$

$$q_{w,2} = \dot{m}_2 \cdot c_{p,2}$$

Derived quantities

Ratio of heat capacity rates $\quad p = \dfrac{q_{w,2}}{q_{w,1}}$

Difference of inlet temperatures $\quad \Delta_0 = T_{1,in} - T_{2,in}$

Calculation of outlet temperatures

$$T_{1,out} = T_{1,in} - p \cdot \Delta_0 \cdot \Psi$$
$$T_{2,out} = T_{2,in} + \Delta_0 \cdot \Psi$$

Co-current

$$P = \frac{1}{q_{w,1}} + \frac{1}{q_{w,2}}$$

$$\Psi_P = \frac{1}{1+p}\left(1 - e^{-U \cdot P \cdot A}\right)$$

Counter-current

$$C = \frac{1}{q_{w,1}} - \frac{1}{q_{w,2}}$$

$$\Psi_C = \frac{1 - e^{-U \cdot C \cdot A}}{p - e^{-U \cdot C \cdot A}}, \text{ if } p \neq 1$$

$$\Psi_C = \frac{\dfrac{U \cdot A}{q_w}}{1 + \dfrac{U \cdot A}{q_w}}, \text{ if } p = 1$$

Problem 14

6 m^3/h 20°C nitric acid solution is to be warmed up. 3 m^3/h 100°C water is available. There is also a double pipe heat exchanger with outer tube of 45/50 mm, inner tube of 25/30 mm, heat conductivity 58 W/mK. Nitric acid solution will flow in the shell. The overall heat transfer coefficient is 2563 W/m^{2}K.

Material properties at average temperatures

	HNO_3 solution	water
ρ [kg/m^3]	1355	965
η [mPas]	2,2	0,316
c_p [J/kgK]	2677	4180
λ [W/mK]	0.5	0.585

Determine the outlet temperatures and the heat power in case of

a) co-currency
b) counter-currency

Note: Overall heat transfer coefficient is given in this problem to make the example shorter but it can also be calculated with the given data.

Solution

Basic quantities

Heat transfer area

$$A = \frac{d_{inner} + d_{outer}}{2} \cdot \pi \cdot L = \frac{2.5 \cdot 10^{-2} m + 3 \cdot 10^{-2} m}{2} \cdot \pi \cdot 15m = 1.3m^2$$

Heat capacity rates

$$q_{w,1} = \dot{m}_1 \cdot c_{p,1} = \dot{V}_1 \cdot \rho_1 \cdot c_{p,1} = \frac{3\frac{m^3}{h}}{3600\frac{s}{h}} \cdot 965\frac{kg}{m^3} \cdot 4180\frac{J}{kg \cdot K} = 3361\frac{W}{K}$$

$$q_{w,2} = \dot{m}_2 \cdot c_{p,2} = \dot{V}_2 \cdot \rho_2 \cdot c_{p,2} = \frac{6\frac{m^3}{h}}{3600\frac{s}{h}} \cdot 1355\frac{kg}{m^3} \cdot 2677\frac{J}{kg \cdot K} = 6046\frac{W}{K}$$

Derived quantities

$$p = \frac{q_{w,2}}{q_{w,1}} = \frac{6046\frac{W}{K}}{3361\frac{W}{K}} = 1.8$$

$$\Delta_0 = T_{1,in} - T_{2,in} = 100°C - 20°C = 80°C$$

a) Co-currency

$$S=\frac{1}{q_{w,1}}+\frac{1}{q_{w,2}}=\frac{1}{3361\frac{W}{K}}+\frac{1}{6046\frac{W}{K}}=4.63\cdot 10^{-4}\ \frac{K}{W}$$

$$\Psi_P=\frac{1}{1+p}\cdot\left(1-e^{-U\cdot P\cdot A}\right)=\frac{1}{1+1.8}\cdot\left(1-e^{-2563\frac{W}{m^2\cdot K}\cdot\left(4.63\cdot 10^{-4}\frac{K}{W}\right)\cdot 1.3m^2}\right)=0.281$$

The same by reading the plot:
$$\left.\begin{array}{l}\dfrac{U\cdot A}{q_{w,1}}=\dfrac{2563\frac{W}{m^2\cdot K}\cdot 1.3m^2}{3361\frac{W}{K}}=1\\ p=1.8\end{array}\right\}\rightarrow \Psi_P=0.28$$

Outlet temperatures

$$T_{1,out}=T_{1,in}-p\cdot\Delta_0\cdot\Psi_P=100°C-1.8\cdot 80°C\cdot 0.281=59.54°C$$

$$T_{2,out}=T_{2,in}+\Delta_0\cdot\Psi_P=20°C+80°C\cdot 0.281=42.48°C$$

Heat power

$$\dot{Q}=\dot{m}_1\cdot c_{p,1}\cdot\left(T_{1,out}-T_{1,in}\right)=\dot{V}_1\cdot\rho_1\cdot c_{p,1}\cdot\left(T_{1,out}-T_{1,in}\right)$$

$$\dot{Q}=\frac{3\frac{m^3}{h}}{3600\frac{s}{h}}\cdot 965\frac{kg}{m^3}\cdot 4180\frac{J}{kg\cdot K}\cdot\left(100°C-59.54°C\right)=1.36\cdot 10^5\,W$$

b) Counter-currency

$$C=\frac{1}{q_{w,1}}-\frac{1}{q_{w,2}}=\frac{1}{3361\frac{W}{K}}-\frac{1}{6046\frac{W}{K}}=1.32\cdot 10^{-4}\ \frac{K}{W}$$

$$\Psi_C=\frac{1-e^{-U\cdot C\cdot A}}{p-e^{-U\cdot C\cdot A}}=\frac{1-e^{-2563\frac{W}{m^2\cdot K}\cdot 1.32\cdot 10^{-4}\frac{K}{W}\cdot 1.3m^2}}{1.8-e^{-2563\frac{W}{m^2\cdot K}\cdot 1.32\cdot 10^{-4}\frac{K}{W}\cdot 1.3m^2}}=0.308$$

The same by reading the plot:
$$\left.\begin{array}{l}\dfrac{U\cdot A}{q_{w,1}}=1\\ p=1.8\end{array}\right\}\rightarrow \Psi_C=0.3$$

Outlet temperatures

$$T_{1,out}=T_{1,in}-p\cdot\Delta_0\cdot\Psi_C=100°C-1.8\cdot 80°C\cdot 0.308=55.65°C$$

$$T_{2,out}=T_{2,in}+\Delta_0\cdot\Psi_C=20°C+80°C\cdot 0.308=44.64°C$$

Heat power

$$\dot{Q}=\dot{m}_1\cdot c_{p,1}\cdot\left(T_{1,in}-T_{1,out}\right)=\dot{V}_1\cdot\rho_1\cdot c_{p,1}\cdot\left(T_{1,in}-T_{1,out}\right)$$

$$\dot{Q}=\frac{3\frac{m^3}{h}}{3600\frac{s}{h}}\cdot 965\frac{kg}{m^3}\cdot 4180\frac{J}{kg\cdot K}\cdot\left(100°C-55.65°C\right)=1.49\cdot 10^5\,W$$

Problem 15

The hot stream enters at 80°C, with 2000 kg/h and 3.14 kJ/kgK. The cold stream enters at 15°C, with 3750 kg/h and 4.18 kJ/kgK. Overall heat transport coefficient is 872 W/m^{2}K. Heat transfer area is 2 m^2.

a) Compute the outlet temperatures and heat powers assuming co-current and counter-current arrangements.
b) How much percentage does the heat power change, both in co-current and counter-current arrangements, if the area is increased to 4 m^2?
c) And if the area is increased to 6 m^2?

Solution

Notation

$T_{1,in}$ = 80°C
$\dot{m}_1$ = 2000 kg/h
$c_{p,1}$ = 3140 J/kgK
$T_{2,in}$ = 15°C
$\dot{m}_2$ = 3750 kg/h
$c_{p,2}$ = 4180 J/kgK
U = 872 W/m^{2}K
A = 2 m^2

a) Compute the outlet temperatures and heat powers assuming co-current and counter-current arrangements.

Basic quantities

Heat capacity rates

$$q_{w,1} = \dot{m}_1 \cdot c_{p,1} = \frac{2000\frac{kg}{h}}{3600\frac{s}{h}} \cdot 3140\frac{J}{kg \cdot K} = 1744.44\frac{W}{K}$$

$$q_{w,2} = \dot{m}_2 \cdot c_{p,2} = \frac{3750\frac{kg}{h}}{3600\frac{s}{h}} \cdot 4180\frac{J}{kg \cdot K} = 4354.17\frac{W}{K}$$

Derived quantities

$$p = \frac{q_{w,2}}{q_{w,1}} = \frac{4354.17\frac{W}{K}}{1744.44\frac{W}{K}} = 2.5$$

$$\Delta_0 = T_{1,in} - T_{2,in} = 80°C - 15°C = 65°C$$

Co-current arrangement

$$P = \frac{1}{q_{w,1}} + \frac{1}{q_{w,2}} = \frac{1}{1744.44\,\frac{W}{K}} + \frac{1}{4354.17\,\frac{W}{K}} = 8.03 \cdot 10^{-4}\,\frac{K}{W}$$

$$\Psi_P = \frac{1}{1+p} \cdot \left(1 - e^{-U \cdot P \cdot A}\right) = \frac{1}{1+2.5} \cdot \left(1 - e^{-872\frac{W}{m^2 \cdot K} \cdot \left(8.03 \cdot 10^{-4}\,\frac{K}{W}\right) \cdot 2m^2}\right) = 0.215$$

Outlet temperatures

$$T_{1,out} = T_{1,in} - p \cdot \Delta_0 \cdot \Psi_P = 80°C - 2.5 \cdot 65°C \cdot 0.215 = 45.06°C$$

$$T_{2,out} = T_{2,in} + \Delta_0 \cdot \Psi_C = 15°C + 65°C \cdot 0.215 = 28.98°C$$

Heat power

$$\dot{Q} = \dot{m}_1 \cdot c_{p,1} \cdot \left(T_{1,in} - T_{1,out}\right) = \frac{2000\,\frac{kg}{h}}{3600\,\frac{s}{h}} \cdot 3140\,\frac{J}{kg \cdot K} \cdot \left(80°C - 45.06°C\right) = 6.095 \cdot 10^4\,W$$

Counter-current arrangement

$$C = \frac{1}{q_{w,1}} - \frac{1}{q_{w,2}} = \frac{1}{1744.44\,\frac{W}{K}} - \frac{1}{4354.17\,\frac{W}{K}} = 3.44 \cdot 10^{-4}\,\frac{K}{W}$$

$$\Psi_C = \frac{1 - e^{-U \cdot C \cdot A}}{p - e^{-U \cdot C \cdot A}} = \frac{1 - e^{-872\frac{W}{m^2 \cdot K} \cdot 3.44 \cdot 10^{-4}\frac{K}{W} \cdot 2m^2}}{2.5 - e^{-872\frac{W}{m^2 \cdot K} \cdot 3.44 \cdot 10^{-4}\frac{K}{W} \cdot 2m^2}} = 0.231$$

Outlet temperatures

$$T_{1,out} = T_{1,in} - p \cdot \Delta_0 \cdot \Psi_C = 80°C - 2.5 \cdot 65°C \cdot 0.231 = 42.46°C$$

$$T_{2,out} = T_{2,in} + \Delta_0 \cdot \Psi_C = 15°C + 65°C \cdot 0.231 = 30.02°C$$

Heat power

$$\dot{Q} = \dot{m}_1 \cdot c_{p,1} \cdot \left(T_{1,in} - T_{1,out}\right) = \frac{2000\,\frac{kg}{h}}{3600\,\frac{s}{h}} \cdot 3140\,\frac{J}{kg \cdot K} \cdot \left(80°C - 42.46°C\right) = 6.55 \cdot 10^4\,W$$

b) How much percentage does the heat power change, both in co-current and counter-current arrangements, if the area is increased to 4 m^2?

Co-current arrangement

$$\Psi'_P = \frac{1}{1+p}\cdot\left(1-e^{-U\cdot P\cdot A'}\right) = \frac{1}{1+2.5}\cdot\left(1-e^{-872\frac{W}{m^2\cdot K}\cdot\left(8.03\cdot 10^{-4}\frac{K}{W}\right)\cdot 4m^2}\right) = 0.268$$

Outlet temperatures

$$T'_{1,out} = T_{1,in} - p\cdot\Delta_0\cdot\Psi'_P = 80°C - 2.5\cdot 65°C\cdot 0.268 = 36.45°C$$

$$T'_{2,out} = T_{2,in} + \Delta_0\cdot\Psi'_C = 15°C + 65°C\cdot 0.268 = 32.42°C$$

Heat power

$$\dot{Q}' = \dot{m}_1\cdot c_{p,1}\cdot\left(T_{1,in} - T'_{1,out}\right) = \frac{2000\frac{kg}{h}}{3600\frac{s}{h}}\cdot 3140\frac{J}{kg\cdot K}\cdot\left(80°C - 36.45°C\right) = 7.6\cdot 10^4\,W$$

Comparison

$$\frac{\dot{Q}'}{\dot{Q}} = \frac{7.6\cdot 10^4\,W}{6.095\cdot 10^4\,W} = 1.247$$

The heat power increases with 24.7 %.

Counter-current arrangement

$$\Psi'_C = \frac{1-e^{-U\cdot C\cdot A'}}{p-e^{-U\cdot C\cdot A'}} = \frac{1-e^{-872\frac{W}{m^2\cdot K}\cdot 3.44\cdot 10^{-4}\frac{K}{W}\cdot 4m^2}}{2.5-e^{-872\frac{W}{m^2\cdot K}\cdot 3.44\cdot 10^{-4}\frac{K}{W}\cdot 4m^2}} = 0.318$$

Outlet temperatures

$$T'_{1,out} = T_{1,in} - p\cdot\Delta_0\cdot\Psi'_C = 80°C - 2.5\cdot 65°C\cdot 0.318 = 28.33°C$$

$$T'_{2,out} = T_{2,in} + \Delta_0\cdot\Psi'_C = 15°C + 65°C\cdot 0.318 = 35.67°C$$

Heat power

$$\dot{Q}' = \dot{m}_1\cdot c_{p,1}\cdot\left(T_{1,in} - T'_{1,out}\right) = \frac{2000\frac{kg}{h}}{3600\frac{s}{h}}\cdot 3140\frac{J}{kg\cdot K}\cdot\left(80°C - 28.33°C\right) = 9.01\cdot 10^4\,W$$

Comparison

$$\frac{\dot{Q}'}{\dot{Q}} = \frac{9.01\cdot 10^4\,W}{6.55\cdot 10^4\,W} = 1.3756$$

The heat power increases with 37.56 %.

c) How much percentage does the heat power change, both in co-current and counter-current arrangements, if the area is increased to 6 m^2?

Co-current arrangement

$$\Psi''_P = \frac{1}{1+p}\cdot\left(1-e^{-U\cdot P\cdot A''}\right) = \frac{1}{1+2.5}\cdot\left(1-e^{-872\frac{W}{m^2\cdot K}\cdot\left(8.03\cdot 10^{-4}\frac{K}{W}\right)\cdot 6m^2}\right) = 0.281$$

Outlet temperatures

$$T''_{1,out} = T_{1,in} - p\cdot\Delta_0\cdot\Psi''_P = 80°C - 2.5\cdot 65°C\cdot 0.281 = 34.34°C$$

$$T'_{2,out} = T_{2,in} + \Delta_0\cdot\Psi'_C = 15°C + 65°C\cdot 0.318 = 35.67°C$$

Heat power

$$\dot{Q}'' = \dot{m}_1\cdot c_{p,1}\cdot\left(T_{1,in} - T''_{1,out}\right) = \frac{2000\frac{kg}{h}}{3600\frac{s}{h}}\cdot 3140\frac{J}{kg\cdot K}\cdot\left(80°C - 34.34°C\right) = 7.97\cdot 10^4\,W$$

Comparison

$$\frac{\dot{Q}''}{\dot{Q}} = \frac{7.97\cdot 10^4\,W}{6.095\cdot 10^4\,W} = 1.308$$

The heat power increases with 30.8 % from the original value.

Counter-current arrangement

$$\Psi''_C = \frac{1-e^{-U\cdot C\cdot A''}}{p-e^{-U\cdot C\cdot A''}} = \frac{1-e^{-872\frac{W}{m^2\cdot K}\cdot 3.44\cdot 10^{-4}\frac{K}{W}\cdot 6m^2}}{2.5-e^{-872\frac{W}{m^2\cdot K}\cdot 3.44\cdot 10^{-4}\frac{K}{W}\cdot 6m^2}} = 0.358$$

Outlet temperatures

$$T''_{1,out} = T_{1,in} - p\cdot\Delta_0\cdot\Psi''_C = 80°C - 2.5\cdot 65°C\cdot 0.358 = 21.83°C$$

$$T''_{2,out} = T_{2,in} + \Delta_0\cdot\Psi''_C = 15°C + 65°C\cdot 0.358 = 38.27°C$$

Heat power

$$\dot{Q}'' = \dot{m}_1\cdot c_{p,1}\cdot\left(T_{1,in} - T''_{1,out}\right) = \frac{2000\frac{kg}{h}}{3600\frac{s}{h}}\cdot 3140\frac{J}{kg\cdot K}\cdot\left(80°C - 21.83°C\right) = 1.01\cdot 10^5\,W$$

Comparison

$$\frac{\dot{Q}''}{\dot{Q}} = \frac{1.01\cdot 10^5\,W}{6.55\cdot 10^4\,W} = 1.542$$

The heat power increases with 54.2 % from the original value.